19.52. **Model:** The heat engine follows a closed cycle with process 1 → 2 and process 3 → 4 being isochoric and process 2 → 3 and process 4 → 1 being isobaric. For a monatomic gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$.

Visualize: Please refer to Figure P19.52.

Solve: **(a)** The first law of thermodynamics is $Q = \Delta E_{th} + W_S$. For the isochoric process 1 → 2, $W_{S\,1\to 2} = 0$ J. Thus,

$$Q_{1\to 2} = 3750 \text{ J} = \Delta E_{th} = nC_V\Delta T$$

$$\Rightarrow \Delta T = \frac{3750 \text{ J}}{nC_V} = \frac{3750 \text{ J}}{(1.0 \text{ mol})\left(\frac{3}{2}R\right)} = \frac{3750 \text{ J}}{(1.0 \text{ mol})\left(\frac{3}{2}\right)(8.31 \text{ J / mol K})} = 301 \text{ K}$$

$$\Rightarrow T_2 - T_1 = 300.8 \text{ K} \Rightarrow T_2 = 300.8 \text{ K} + 300 \text{ K} = 601 \text{ K}$$

To find volume V_2,

$$V_2 = V_1 = \frac{nRT_1}{p_1} = \frac{(1.0 \text{ mol})(8.31 \text{ J / mol K})(300 \text{ K})}{3.0\times 10^5 \text{ Pa}} = 8.31\times 10^{-3} \text{ m}^3$$

The pressure p_2 can be obtained from the isochoric condition as follows:

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} \Rightarrow p_2 = \frac{T_2}{T_1}p_1 = \left(\frac{601 \text{ K}}{300 \text{ K}}\right)\left(3.00\times 10^5 \text{ Pa}\right) = 6.01\times 10^5 \text{ Pa}$$

With the above values of p_2, V_2 and T_2, we can now obtain p_3, V_3 and T_3. We have

$$V_3 = 2V_2 = 1.662\times 10^{-2} \text{ m}^3 \qquad p_3 = p_2 = 6.01\times 10^5 \text{ Pa}$$

$$\frac{T_3}{V_3} = \frac{T_2}{V_2} \Rightarrow T_3 = \frac{V_3}{V_2}T_2 = 2T_2 = 1202 \text{ K}$$

For the isobaric process 2 → 3,

$$Q_{2\to 3} = nC_P\Delta T = (1.0 \text{ mol})\left(\tfrac{5}{2}R\right)(T_3 - T_2) = (1.0 \text{ mol})\left(\tfrac{5}{2}\right)(8.31 \text{ J / mol K})(601 \text{ K}) = 12{,}480 \text{ J}$$

$$W_{S\;2\to 3} = p_3(V_3 - V_2) = \left(6.01\times 10^5 \text{ Pa}\right)\left(8.31\times 10^{-3} \text{ m}^3\right) = 4990 \text{ J}$$

$$\Delta E_{th} = Q_{2\to 3} - W_{S\;2\to 3} = 12{,}480 \text{ J} - 4990 \text{ J} = 7490 \text{ J}$$

We are now able to obtain p_4, V_4 and T_4. We have

$$V_4 = V_3 = 1.662\times 10^{-2} \text{ m}^3 \qquad p_4 = p_1 = 3.00\times 10^5 \text{ Pa}$$

$$\frac{T_4}{p_4} = \frac{T_3}{p_3} \Rightarrow T_4 = \frac{p_4}{p_3}T_3 = \left(\frac{3.00\times 10^5 \text{ Pa}}{6.01\times 10^5 \text{ Pa}}\right)(1202 \text{ K}) = 600 \text{ K}$$

For isochoric process 3 → 4,

$$Q_{3\to 4} = nC_V\Delta T = (1.0 \text{ mol})\left(\tfrac{3}{2}R\right)(T_4 - T_3) = (1.0 \text{ mol})\left(\tfrac{3}{2}\right)(8.31 \text{ J / mol K})(-602) = -7500 \text{ J}$$

$$W_{S\;3\to 4} = 0 \text{ J} \Rightarrow \Delta E_{th} = Q_{3\to 4} - W_{S\;3\to 4} = -7500 \text{ J}$$

For isobaric process 4 → 1,

$$Q_{4\to 1} = nC_P\Delta T = (1.0 \text{ mol})\tfrac{5}{2}(8.31 \text{ J / mol K})(300 \text{ K} - 600 \text{ K}) = -6230 \text{ J}$$

$$W_{S\;4\to 1} = p_4(V_1 - V_4) = \left(3.00\times 10^5 \text{ Pa}\right)\times\left(8.31\times 10^{-3} \text{ m}^3 - 1.662\times 10^{-2} \text{ m}^3\right) = -2490 \text{ J}$$

$$\Delta E_{th} = Q_{4\to 1} - W_{S\;4\to 1} = -6230 \text{ J} - (-2490 \text{ J}) = -3740 \text{ J}$$

	W_S (J)	Q (J)	ΔE_{th} (J)
1 → 2	0	3750	3750
2 → 3	4990	12,480	7490
3 → 4	0	–7500	–7500
4 → 1	–2490	–6230	–3740
Net	2500	2500	0

(b) The thermal efficiency of this heat engine is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{H}}} = \frac{W_{\text{out}}}{Q_{1\to 2} + Q_{2\to 3}} = \frac{2500\text{ J}}{3750\text{ J} + 12{,}480\text{ J}} = 0.154 = 15.4\%$$

Assess: For a closed cycle, as expected, $(W_{\text{s}})_{\text{net}} = Q_{\text{net}}$ and $(\Delta E_{\text{th}})_{\text{net}} = 0\text{ J}$